Name: Date: Period: 6

Law of Sines and Law of Cosines

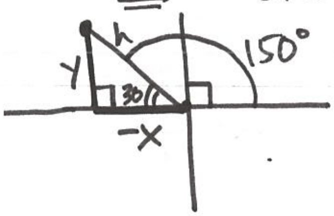
Lesson Objective IWBAT use the Law of Sines and Law of Cosines to solve non-right Δs.

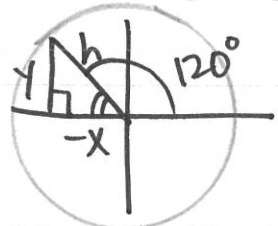
So far, you have used trigonometric ratios to solve right triangles. In this lesson objective, you will learn how to solve ANY triangle. When the triangle is obtuse, you may need to find a trigonometric ratio for an obtuse angle.

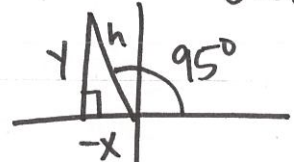
Example

1. Use a calculator to find each trigonometric ratio. Round your answer to three decimal places.

a. $\tan 150° \approx -0.577$ b. $\sin 120° \approx 0.866$ c. $\cos 95° \approx -0.087$

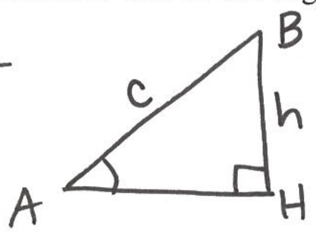






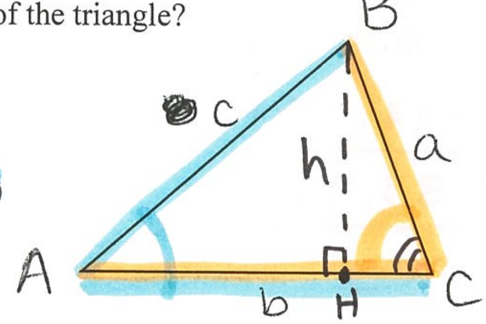
Think About It…How can we use trig ratios to find the *height* of the triangle?

$A=\frac{bh}{2}$



$\frac{\sin A}{1}=\frac{h}{c}$

$h=(c)(\sin A)$



$\frac{\sin C}{1}=\frac{h}{a}$

$h=(a)(\sin C)$

Then, how can we use it to find the area of the triangle?

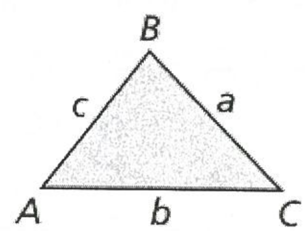
$\text{area}=\frac{bh}{2}$

$=\frac{(b)(c)(\sin A)}{2}$

$\text{area}=\frac{(b)(a)(\sin C)}{2}$

Area of a Triangle (Using Trig Ratios)

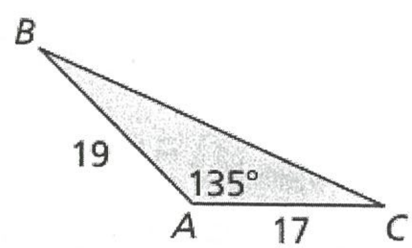
The area of any triangle is given by one-half the product of the lengths of two sides times the sine of their included angle. For ΔABC shown, there are three ways to calculate the area…



$\text{area}=\frac{a\cdot b\cdot \sin C}{2}$ OR $=\frac{b\cdot c\cdot \sin A}{2}$ OR $=\frac{a\cdot c\cdot \sin B}{2}$

Example:

2. Find the area of the triangle. Round your answer to three decimal places.



$\text{area}=\frac{(19)(17)(\sin 135°)}{2}$

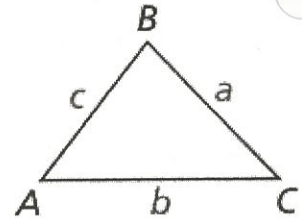
$=(323)(0.707)$ $\boxed{\approx 114.198 \text{ units}^2}$

Law of Sines

You can use the Law of Sines to solve triangles when…

1. two angles and the length of any side are known (AAS or ASA cases)
2. when the lengths of two sides and an angle opposite one of those sides are known (SSA case)

Law of Sines (Theorem)

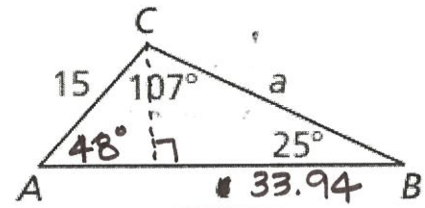


The Law of Sines can be written in either of the following forms for ΔABC with side lengths a (opposite $\angle A$), b (opposite $\angle B$) and c (opposite $\angle C$)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example:

3. Solve the triangle. Round answers to two decimal places. (AAS)



$$\frac{c}{\sin 107} = \frac{15}{\sin 25}$$

$$\frac{c}{0.9563} = \frac{15}{0.4226}$$

$$0.4226c = 14.3445$$

$$\boxed{c \approx 33.94}$$

$\boxed{m\angle A = 48°}$ by Δ sum thm

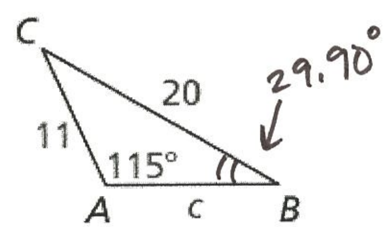
$$\frac{a}{\sin 48} = \frac{15}{\sin 25}$$

$$\frac{a}{0.7431} = \frac{15}{0.4226}$$

$$0.4226a = 11.1465$$

$$\boxed{a \approx 26.38}$$

4. Solve the triangle. Round answers to two decimal places.



(SSA - solve for missing ∠ opp. the given side first)

① $$\frac{11}{\sin B} = \frac{20}{\sin 115}$$

$$\frac{11}{\sin B} = \frac{20}{0.9063}$$

$$20(\sin B) = 9.9694$$

$$\sin B = 0.4985$$

$$m\angle B = \sin^{-1}(0.4985) \boxed{\approx 29.90°}$$

② $\boxed{m\angle C \approx 35.1°}$ by Δ sum thm

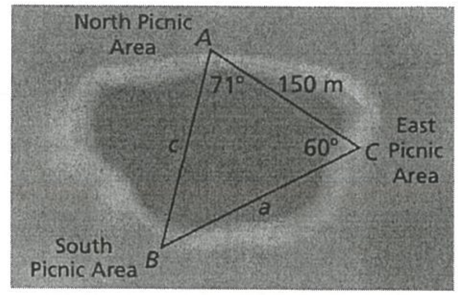
③ $$\frac{20}{\sin 115} = \frac{c}{\sin 35.1}$$

$$\frac{20}{0.9063} = \frac{c}{0.5750}$$

$$0.9063c = 11.5001$$

$$\boxed{c \approx 12.69}$$

5. A surveyor makes the measurements shown to determine the length of a bridge to be built across a small lake from the North Picnic Area to the South Picnic Area. Find the length of the bridge.



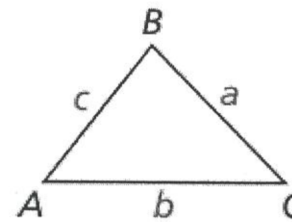
Law of Cosines

You can use the Law of Cosines to solve triangles when…

1. Two sides and the included angle are known (SAS case)
2. All three sides are known (SSS case)

Law of Cosines (Theorem)

The Law of Cosines can be written in either of the following forms for ΔABC with side lengths a (opposite $\angle A$), b (opposite $\angle B$) and c (opposite $\angle C$)

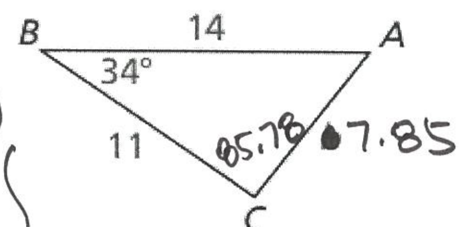


$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

"PEMDAS"

Example:

6. Solve the triangle. Round answers to two decimal places.



(SAS – solve for side across from given ∠ first)

① $b^2 = [(11)^2 + (14)^2] - [2(11)(14)(\cos 34)]$

$b^2 = 317 - [(308)(0.8290)]$

$b^2 = 317 - 255.3436$

$\sqrt{b^2} = \sqrt{61.6564}$

$\boxed{b \approx 7.85}$

③ $\boxed{m\angle A \approx 60.22°}$

by Δ sum thm

② $\dfrac{14}{\sin C} = \dfrac{7.85}{\sin 34}$

$\dfrac{14}{\sin C} = \dfrac{7.85}{0.5592}$

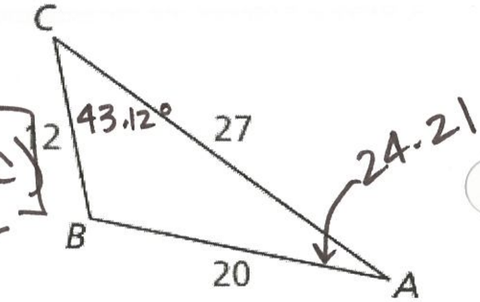
$7.85(\sin C) = 7.828$

$\sin C = 0.9973$

$m\angle C = \sin^{-1}(0.997$

$\boxed{m\angle C \approx 85.78°}$

7. Solve the triangle. Round answers to two decimal places.



(SSS – solve for any ∠ first)

① $20^2 = [(12)^2 + (27)^2] - [2(12)(27)(\cos C)]$

$400 = 873 - [648(\cos C)]$

$-473 = -648(\cos C)$

$\cos C = 0.7299$

$m\angle C = \cos^{-1}(0.7299)$

$\boxed{m\angle C \approx 43.12°}$

② $\dfrac{12}{\sin A} = \dfrac{20}{\sin 43.12}$

$\dfrac{12}{\sin A} = \dfrac{20}{0.68356}$

$20(\sin A) = 8.2026$

$\sin A = 0.4101$

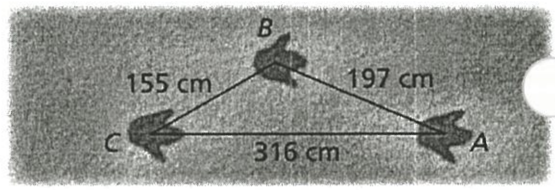
$m\angle A = \sin^{-1}(0.4101)$

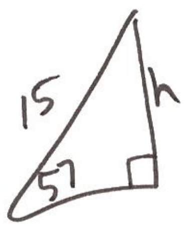
$\boxed{m\angle A \approx 24.21°}$

③ $\boxed{m\angle B \approx 112.67°}$

by Δ sum thm

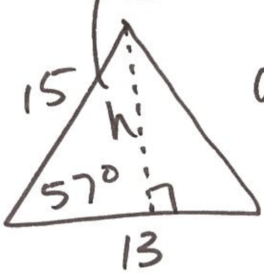
8. An organism's step angle is a measure of walking efficiency. The closer the step angle is to 180°, the more efficiently the organism walked. The diagram shows a set of footprints for a dinosaur. Find the step angle ($m\angle B$).





$\sin 57 = \dfrac{h}{15}$

$h = (15)(\sin 57)$



area $= \dfrac{(15)(13)(\sin 57)}{2}$

$\boxed{\approx 81.77 \text{ units}^2}$